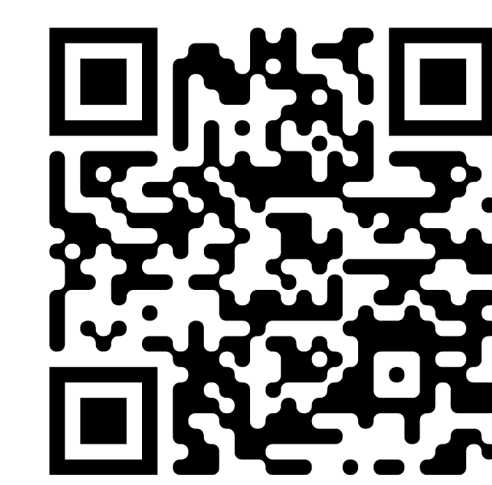
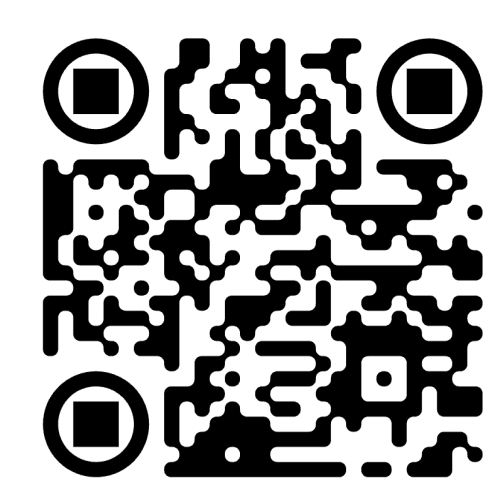




LinkedIn Davide



LinkedIn Andrea

What is ESG-Constrained Portfolio Optimisation?

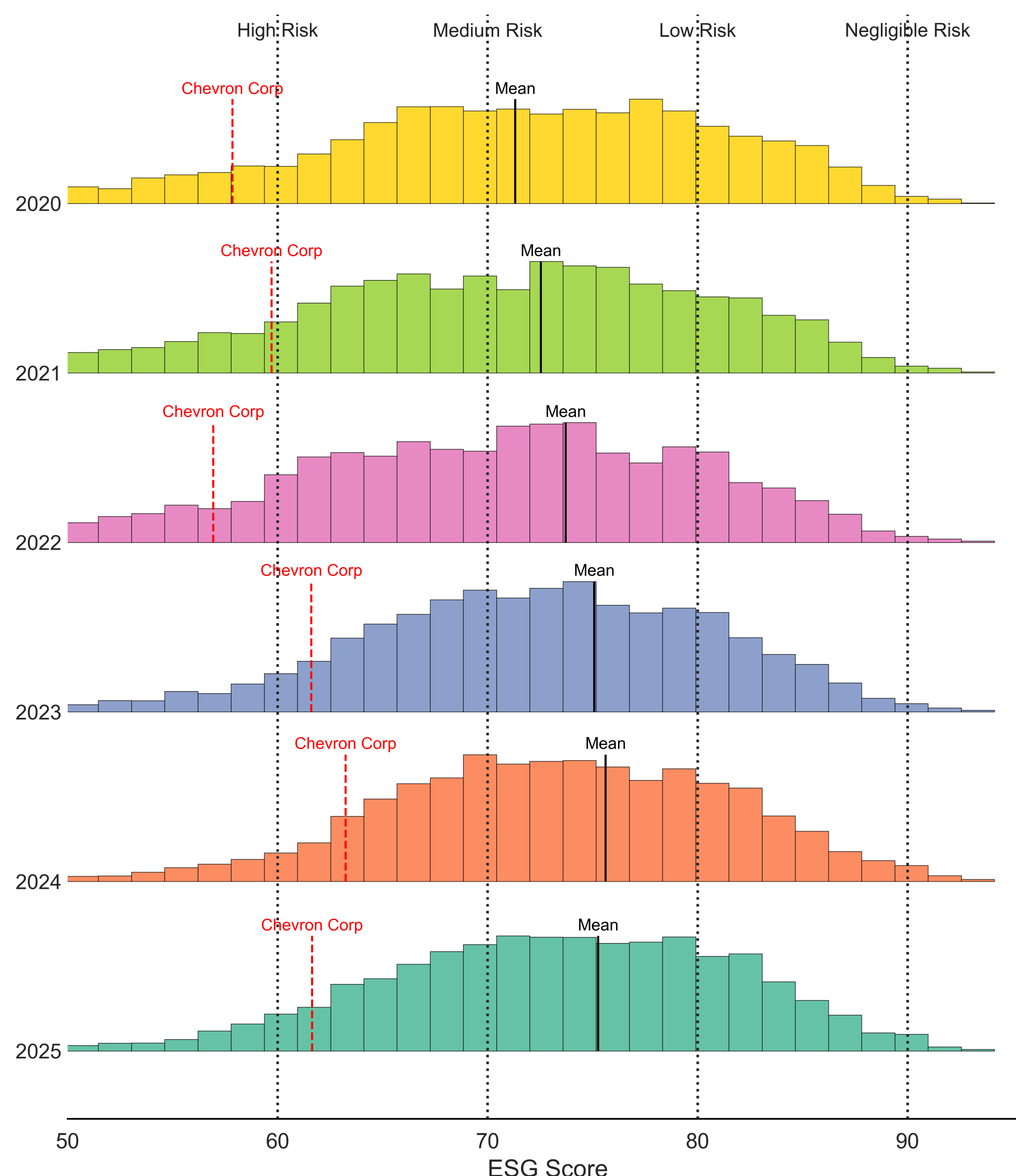


Figure 1. ESG score distribution across years. Chevron Corp exhibits persistent high ESG risk.

- ESG scores reflect environmental, social, and governance sustainability of firms.
- High-risk companies (e.g., Chevron) score significantly below the market average.
- An ESG-aware investor seeks to constrain allocations to maintain sustainability exposure.

Market Dynamics and Wealth Evolution

Wealth process with ESG-constrained allocation:

$$\frac{dX_t}{X_t} = (r_t + \omega_{A,t}(\mu_{A,t} - r_t) + \omega_t^\top (\mu_t - r_t \mathbf{1}))dt + (\omega_{A,t} \sigma_{A,t}^\top + \omega_t^\top \Sigma_t^\top) dW_t \quad (1)$$

ESG Linear Constraint:

$$\lambda_t - \psi_{G,t} - \omega_{A,t}(\psi_{A,t} - \psi_{G,t}) - \omega_t^\top (\psi_{S,t} - \psi_{G,t} \mathbf{1}) = 0 \quad (2)$$

Objective Function and Optimisation Problem

$$\max_{\omega_t} \mathbb{E}_t \left[\frac{X_T^{1-\delta}}{1-\delta} e^{-\int_t^T \rho_s ds} \right] \quad \text{s.t. ESG constraint} \quad (3)$$

$$\max_{\omega_t} \mathbb{E}_t^{\text{ESG}} \left[\frac{X_T^{1-\delta}}{1-\delta} e^{-\int_t^T \rho_s ds} \right] \quad \text{s.t. ESG constraint} \quad (4)$$

Optimal Solutions

Using HJB, Martingale and OLS approximation we obtained:

Under the physical measure $\mathbb{P}$:

$$\omega_t = (\eta_t^\top \eta_t)^{-1} \eta_t^\top \left(\frac{1}{\delta} \xi_t + \Phi_t \frac{\partial \ln H_{t,z_t}}{\partial z_t} - \frac{\lambda_t - \psi_{G,t}}{\psi_{A,t} - \psi_{G,t}} \sigma_{A,t} \right) \quad (5)$$

Under ESG-constrained beliefs (subjective measure):

$$\omega_t^{\text{ESG}} = (\eta_t^\top \eta_t)^{-1} \eta_t^\top \left(-\frac{1}{\delta} \nu_t + \Phi_t \frac{\partial \ln F_{t,z_t}}{\partial z_t} - \frac{\lambda_t - \psi_{G,t}}{\psi_{A,t} - \psi_{G,t}} \sigma_{A,t} \right) \quad (6)$$

ESG Constraint and Optimal Portfolios

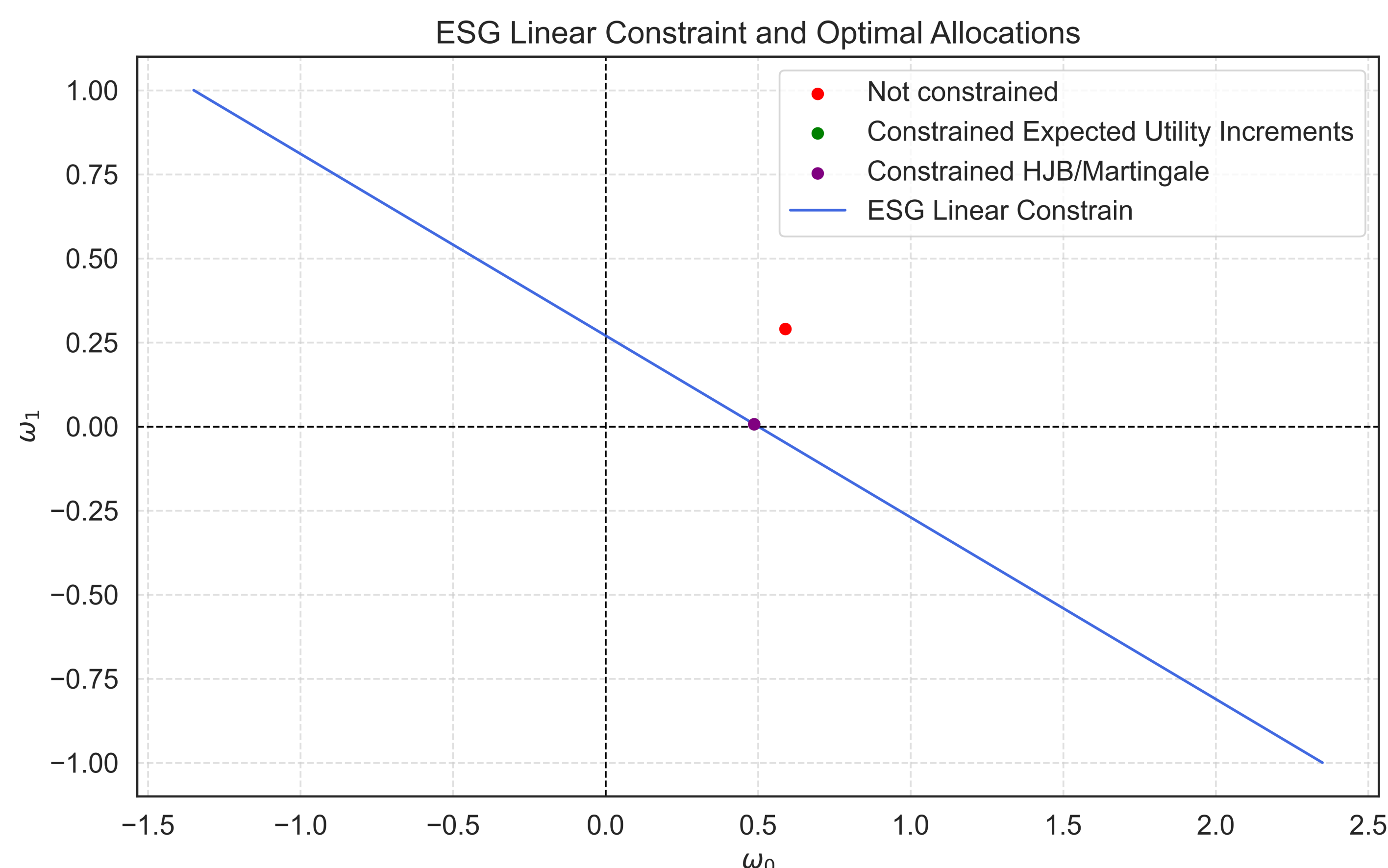


Figure 2. Linear ESG constraint and optimal allocations: unconstrained (red), constrained (green, purple)

Expected Utility and Penalisation

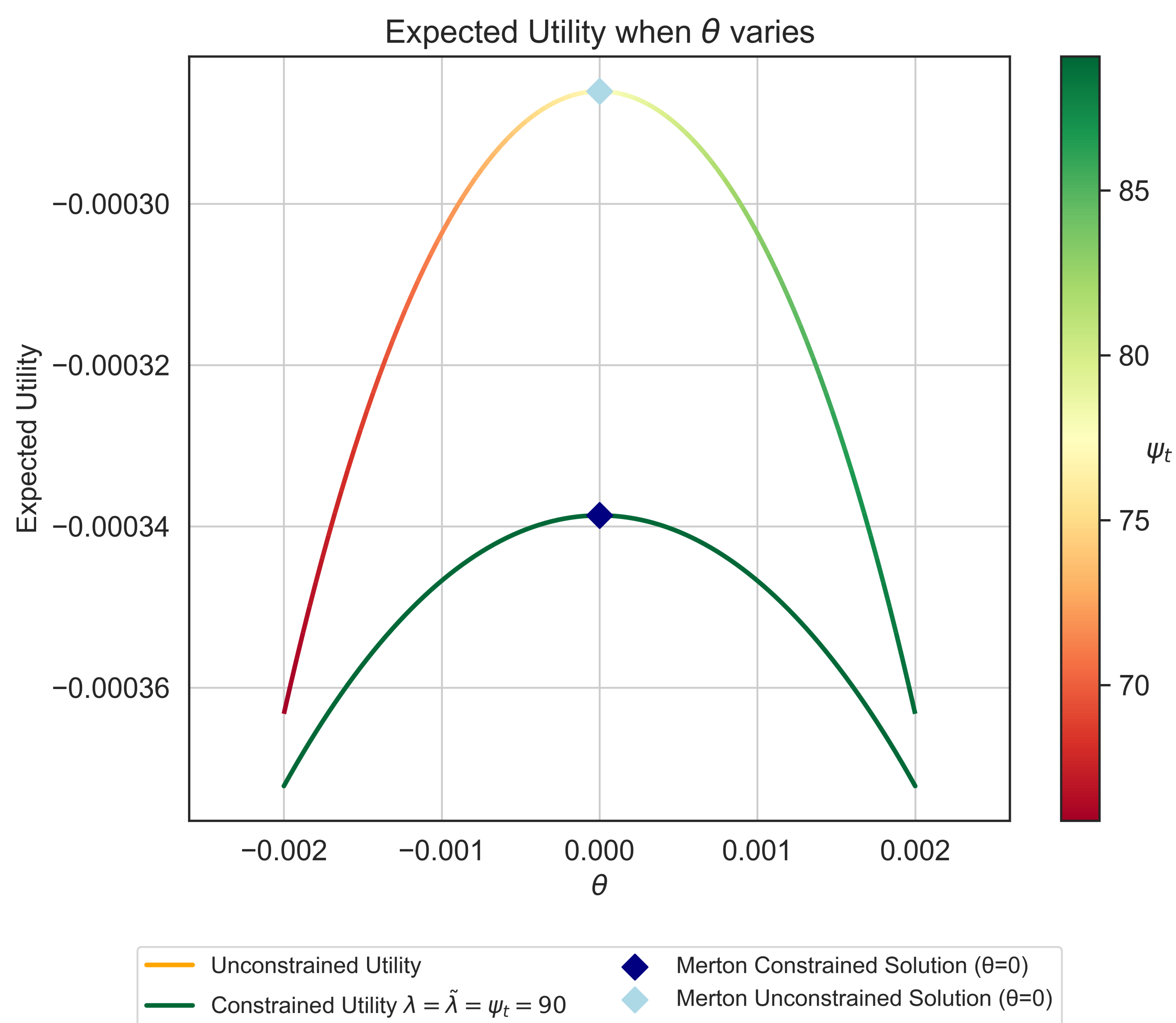


Figure 3. Expected utility of wealth when the penalisation coefficient θ varies.

Interpretation and Results

- **Vertical gap** between red and dark blue = utility loss from ESG constraint.
- **Horizontal shift** along green = subjective correction via θ penalisation.
- The optimal point balances ESG alignment with financial efficiency.

Main Takeaways

- ESG preferences can be included as constraints or distortions in the probability space.
- Closed-form approximations and constrained allocations are derived.
- The method supports both **theoretical insight** and **empirical calibration**.
- The framework is extended to **stochastic ESG scores**.