Derivatives in Finance: The Forward Contract Explained

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Introduction to Derivatives

A derivative is a financial instrument whose value is derived from the performance of an underlying asset. The underlying can be a stock, a market index, a commodity, a currency, or virtually any asset with an observable market price. Derivatives are widely used for hedging, speculation, and arbitrage, allowing market participants to manage risk or gain exposure to price movements without directly holding the underlying asset. There are various types of derivatives, but the most commonly used are forward, futures, swaps, and options. These instruments can also be classified based on where and how they are traded:

- Exchange-traded derivatives are standardized contracts traded on regulated exchanges (such as the CME or Eurex). These contracts have fixed terms (e.g., expiration dates, contract sizes) and are cleared through a central clearinghouse, reducing counterparty risk.
- Over-the-counter (OTC) derivatives are privately negotiated agreements between two parties. They are customizable in terms of contract size, maturity, and underlying, but carry higher counterparty risk, as they are not traded on a formal exchange.

Today, we will focus on one of the most fundamental derivative instruments: the forward contract, which is typically traded OTC.

The Forward contract

A forward contract is an agreement between two parties to buy or sell an asset at a specific future date for a price agreed upon today. One key characteristic of a forward contract is that it involves no initial cash flow at the time the contract is signed (known as the inception date). At maturity (denoted as time T), the buyer of the contract pays the forward price F_T, and the seller delivers the underlying asset as specified in the agreement. The forward price is typically set such that the contract has zero value at inception, meaning neither party has an advantage when the deal is made.

Forward contracts are widely used in practice, especially in commodities, currencies, and interest rate markets. Their flexibility allows for tailored agreements, but this also means they are exposed to counterparty default risk and limited liquidity compared to exchange-traded instruments (this limitation is addressed by other derivatives that we will analyse in a future episode).

Let us consider the price of a security $X(t)$ at time t in an arbitrage-free market:

$$X(t) = \mathbb{E}_t^{\mathbb{Q}}\left[X(T)\frac{G(t)}{G(T)}\right]. \tag{1}$$

The equation above represents one of the most fundamental results in modern finance: the *Fundamental Theorem of Finance*. It establishes a general relationship between the present value of an asset and the discounted expected value of its future cash flows under the risk-neutral probability measure $\mathbb{Q}$. In a future episode, we will explain the meaning of discounting cash flows under the risk-neutral measure $\mathbb{Q}$, and what it means to change probability measures. For now, it is sufficient to understand that this relationship provides a powerful tool for asset pricing. When the Fundamental Theorem of Finance holds, it allows us to price both simple and highly complex financial instruments. The key is to focus on the technical features of the asset, particularly the nature and timing of its cash flows, and to express them using the framework provided by the theorem. In this case, we are fortunate, as the pricing of a forward contract is relatively straightforward.

Suppose that today you wish to buy a certain quantity of oil for delivery at a future date T. You do not receive the oil today, but you want to fix its price in advance to avoid exposure to future price fluctuations. In other words, you are looking for a contract that sets the price you will pay at maturity. This is the purpose of a *forward contract*. It allows the buyer to purchase an asset $X(T)$, whose price is stochastic, by paying a predetermined price at time T. At the inception of the contract, the following condition must hold:

$$0 = \mathbb{E}_0^{\mathbb{Q}}\left[(X(T) - F_T)\frac{G(0)}{G(T)}\right], \tag{2}$$

where F_T is a fixed price fixed in $t = 0$ which stays the same until maturity T. As we already highlighted the price of this contract is 0 as there are no cash flows at the starting time (nobody pays anyone). If the price was different from zero today it would mean that one of the two counterparties would accept to have an expected negative (positive) value from this contract. With a little bit of algebra we can rewrite:

$$\begin{aligned}
0 &= \mathbb{E}_0^{\mathbb{Q}}\left[(X(T) - F_T)\frac{G(0)}{G(T)}\right] \\
&= \mathbb{E}_0^{\mathbb{Q}}\left[X(T)\frac{G(0)}{G(T)}\right] - \mathbb{E}_0^{\mathbb{Q}}\left[F_T\frac{G(0)}{G(T)}\right] \\
&= \mathbb{E}_0^{\mathbb{Q}}\left[X(T)\frac{G(0)}{G(T)}\right] - F_T B(0,T).
\end{aligned} \tag{3}$$

This equation allows us to derive a closed-form expression for the fair forward price F_T[1]:

[1] This formula assumes that the underlying asset does not pay dividends or generate intermediate cash flows between 0 and T.

$$F_T = \frac{X(0) - D(0,T)}{B(0,T)}, \tag{4}$$

where δ are cash flows generated by the underlying asset. Moreover, $D(0,T)$ represent the present value of all the dividends, in particular:

$$D(0,T) = \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^T \delta(s)\frac{G(0)}{G(s)}ds\right). \tag{5}$$

In Equation 4, $X(0)$ is the price of the underlying asset at time $t = 0$. From this, we subtract the present value of future cash flows $D(0,T)$, and then divide the result by the price of a zero-coupon bond maturing at time T, denoted by $B(0,T)$. This gives us the fair forward price F_T under an arbitrage-free market. The value of the contract is equal to zero when you seal the contract in t=0 but from that time on it is not necessarily true. The reason why the price fluctuates over time is that F_T is fixed and stays unchanged until the end of the contract while X_T changes in a way that the fixed price might be unfair. The value at any time t is the following:

$$\begin{aligned} F(t,T) &= \mathbb{E}_t^{\mathbb{Q}}\left[(X(T) - F_T)\frac{G(t)}{G(T)}\right] \\ &= \mathbb{E}_t^{\mathbb{Q}}\left[X(T)\frac{G(t)}{G(T)}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[F_T\frac{G(t)}{G(T)}\right] \\ &= X(t) - D(t,T) - F_T B(t,T) \end{aligned} \tag{6}$$

We therefore obtain that the price of the forward contract is zero at inception, $F(0,T) = 0$, and evolves over time according to the relation $F(t,T) = X(t) - D(t,T) - F_T B(t,T)$. This dynamic has a clear economic interpretation: the value of the forward contract at any time t corresponds to the difference between the current spot price of the underlying asset and the present value of the fixed delivery price F_T, discounted to time t. If the asset pays cash flows, the discounted value of all of them $(D(0,T))$ has to be subtracted too. If the forward price F_T, when discounted using the current term structure between t and T, is lower than the present value of the security (minus the dividends), then the forward has positive value for the holder. It means that we are allowed to pay a price at the end which is lower than the real value of the asset. In such a case, the forward contract gives us the right to acquire the asset for less than its current fair value, and the counterparty would need to compensate us for that advantage, just as we would compensate them in the reverse scenario.

Moreover, one can write:

$$F(t,T) = X(t) - D(0,T) - \frac{X(0)}{B(0,T)}B(t,T), \tag{7}$$

This alternative formulation highlights the forward price dynamics more explicitly.

Contango vs Backwardation

We can now introduce the forward curve, which describes how the forward price F_T varies with respect to time to maturity T. Considering 4 and differentiating it with respect to T, we obtain:

$$\frac{\partial F_T}{\partial T} = \frac{\frac{\partial (X(0) - D(0,T))}{\partial T} B\left(0,T\right) - \frac{\partial B(0,T)}{\partial T}\left(X\left(0\right) - D\left(0,T\right)\right)}{B\left(0,T\right)^2}. \tag{8}$$

Using the identity $\left(X\left(0\right) - D\left(0,T\right)\right) = F_T B\left(0,T\right)$, we simplify the expression:

$$\frac{\partial F_T}{\partial T} = \frac{-\mathbb{E}_0^{\mathbb{Q}}\left(\delta\left(T\right)\frac{G(0)}{G(T)}\right) B\left(0,T\right) - \frac{\partial B(0,T)}{\partial T} F_T B\left(0,T\right)}{B\left(0,T\right)^2}, \tag{9}$$

or, more compactly:

$$\frac{\partial F_T}{\partial T} = -\frac{\mathbb{E}_0^{\mathbb{Q}}\left(\delta\left(T\right)\frac{G(0)}{G(T)}\right)}{B\left(0,T\right)} - F_T f\left(0,T\right), \tag{10}$$

where the instantaneous forward rate is defined as:

$$f\left(0,T\right) = -\frac{\partial B\left(0,T\right)}{\partial T}\frac{1}{B\left(0,T\right)}. \tag{11}$$

Finally, the expectation term can be simplified further by introducing a new probability measure $\mathbb{F}$, under which the zero-coupon bond maturing at T becomes the numéraire[2]. Under this measure, we write:

$$\frac{\partial F_T}{\partial T} = -\mathbb{E}_0^{\mathbb{F}}\left(\delta\left(T\right)\right) + F_T f\left(0,T\right), \tag{12}$$

and equivalently:

$$\frac{\partial F_T}{\partial T}\frac{1}{F_T} = f\left(0,T\right) - \frac{\mathbb{E}_0^{\mathbb{F}}\left(\delta\left(T\right)\right)}{F_T}. \tag{13}$$

It is evident that the sign of this derivative depends on the relationship between $f(0,T)$ and $\frac{\mathbb{E}_0^{\mathbb{F}}(\delta(T))}{F_T}$. The latter can be interpreted as a sort of dividend yield price. Two situations can occur:

- If the instantaneous forward rate exceeds the dividend price, then the derivative is positive and F_T increases with T. In this case, the resulting forward curve is said to be in ***contango***.
- On the other hand, if the forward rate is lower than the dividend price, then the derivative is negative and the curve decreases with T. This situation is referred to as ***backwardation***.

These two scenarios are illustrated in Figure 1, which shows the typical curvature of forward price functions under contango and backwardation. It is also worth noting that the sign of the derivative may vary across different maturities. In the special case where the underlying

[2]In this case we can simplify $\mathbb{E}_0^{\mathbb{Q}}\left(\delta\left(T\right)\frac{G(0)}{G(T)}\right) = \mathbb{E}_0^{\mathbb{F}}\left(\delta\left(T\right)\right)\mathbb{E}_0^{\mathbb{Q}}\left(\frac{G(0)}{G(T)}\right) = \mathbb{E}_0^{\mathbb{F}}\left(\delta\left(T\right)\right)B\left(0,T\right)$.

asset does not pay dividends ($\mathbb{E}_0^{\mathbb{F}}[\delta(T)] = 0$), and assuming a positive forward rate $f(0,T)$, the forward curve is strictly increasing and thus always in contango.

In practical terms, forward curves are typically in contango when the cost of carrying or storing the underlying asset is high. For example, in commodity markets like oil or metals, storage costs, insurance, and financing expenses are reflected in higher forward prices. This leads to an upward-sloping curve, as buyers are willing to pay more for future delivery to avoid incurring storage costs themselves.

Conversely, backwardation tends to occur when there is a high convenience yield, that is, a strong benefit to physically holding the asset today rather than in the future. This is common in markets where supply shortages or seasonal demand make immediate delivery more valuable. In such situations, spot prices can be higher than forward prices, resulting in a downward-sloping curve.

Therefore, the shape of the forward curve is influenced by a combination of interest rates, expected dividends or cash flows, storage costs, and convenience yields, and reflects the relative trade-off between owning the asset now versus committing to buy it in the future.

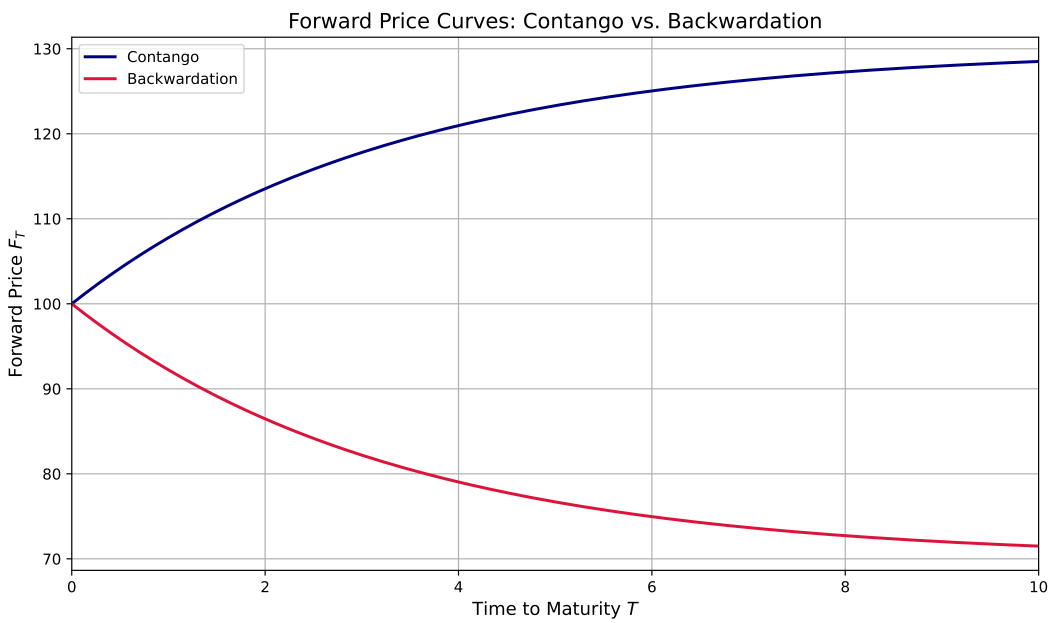

Figure 1: Illustration of forward price curves under contango and backwardation regimes. The x-axis represents the time to maturity T, while the y-axis shows the corresponding forward price F_T. The contango curve (in navy) increases over time. The backwardation curve (in red) declines over time, indicating lower future prices, often associated with high convenience yield or expected supply constraints. Both curves originate from the spot price $X(0) = 100$.